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Multi-Objective Optimization of Planetary Gear Train Using Genetic Algorithm

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A B S T R A C T

Planetary gearboxes are used in different industrial fields such as automobiles, helicopters, heavy machinery and wind turbines. They have many advantages such as compact dimensions, less noise, high gear ratio and higher torque-to-weight ratios, compared to standard parallel axis gearboxes. This paper presents a multi-objective optimization model for minimizing the mass of a planetary gear train and maximizing gear ratio under the constraints of gear teeth bending stress, contact stress and side constraints .The selected design variables are the module, gear teeth width, number of teeth for both sun and planet gears, inner diameter of both sun and planet gears and the outer diameter of the ring gear. Different types of materials for the planetary gears have been studied. The optimization problem has been formulated and solved using the Genetic Algorithm (GA). The obtained results indicated that the optimum mass of the planetary gear train has maximum values in case of stainless steel while has relatively equal values in case of plastic material. The maximum gear ratios were observed in case of 1.1 KW input power

Nomenclature

P_{in}	Input Power of Motor (KW)	K_o	Power Source and Driven Machine Factor
N_{in}	Input Speed (rev/min)	Y_N	Stress Cycle Life
h_s	Gear Tooth Height (m)	Z_N	Pitting Resistance Stress Cycle
α	Pressure Angle	C_p	Elastic Coefficient Factor
a	Center Distance (m)	K_f	Fatigue Concentration Factor
ρ	Density of Gear Material	C_H	Hardness Ratio Factor
K_v	Dynamic Factor	I	Pitting Geometry Factor
Y_j	Modified Lewis Factor	K_R	Reliability Factor
K_a	Application Factor	m	Module
K_s	Size Factor		
K_m	Load Distribution Factor		
K_i	Idler Factor		
K_B	Rim Thickness Factor		

1. Introduction

Among all power transmitting machine elements, gears are the most important mechanical element [1]. Gears are employed in wide range of applications such as aeronautics, automobile, machine tools, wind turbines, children toys and house electrical instruments [2]. The simplest of all gears is the spur gears; they offer considerable precision and high power transmission efficiency [3]. Planetary gear train is a form of gearbox structure

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which consists of four elements. The four elements are mainly; a sun gear, one or more planet gears, a ring gear and an arm (planet carrier) [4] as shown in Figure 1. Sun gear is always located at the center and transmits torque to planet gears orbiting around the sun gear. The planet gears are mounted on an arm or carrier (surrounded by the ring gear) that fixes the planets in an orbit relative to each other [5]. Planetary gears are found in many variations and arrangements in order to meet a broad range of speed ratios in the design requirements. One element of the planetary gear train configuration is fixed and the other elements are rotated according to the input and output members. Different configurations can be easily obtained by re-arranging input member, outer member and stationary member.

Planetary gearboxes have a various application in different mechanical systems, such as industrial drives, rotorcraft, automobiles, wind turbines, etc., where they can offer compact dimensions, less noise, high gear ratio and higher torque-to-weight ratios, especially compared to standard parallel axis gear trains [6, 7, 8]

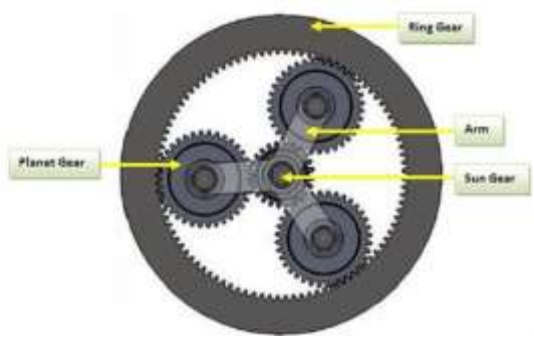

Figure 1: Planetary Gear train

S.B. Nandeppagoudar, et al [9] designed a three stage planetary gear box suitable for machine tool application. Reduction of material has been one the critical aspects of any design along with reduction in deformation and stress factors, which increases the life of the product as presented by Fatmir Azemi, et al [10], authors presented design, analysis and shape optimization in order to reduce material of spur gears and focuses on static analysis. Finite element analysis has been used to implement optimization and maintaining stress and deformation levels. Brahim Mahiddini, et al [11] presented a two level design optimization methodology for a simple reducer. The first level is based on using an analytical formulation for the design problem based on classical gear and beam theory, while the second level was used to construct a fine CAD model of the reducer. In fact, the design of the planetary gear train is highly complicated and involves many aspects. This complexity leads to many design variables, mathematical formulations, constraints, and many influencing factors as studied by Kaoutar Daoudi, et al [12]. They described a multi-objective optimization for the epicyclical gear train system using the GA. The purpose of this study is to minimize the weight and the centre distance of one pair of spur gears. This objective was accomplished by means of the GA under some constraint such as bending strength, contact stress and each dimension conditions of gears, which must be satisfied. The results are calculated by using MATLAB tools of Genetic algorithm for three types of materials, which are alloy steel, cast iron, and epoxy glass composites. Paridhi Rai and Asim Gopal Barman [2], have extended a gear design model to include more AGMA geometrical factors along with some modifications compared to earlier studies. The design optimization of the spur gear set is carried out using two powerful optimization tools, simulated annealing and real-coded genetic algorithm.

2. Gear Geometry and Stresses on Gear Tooth

According to the geometry for the gear profile, the basic dimensions of a gear are verified as shown in Figure 2

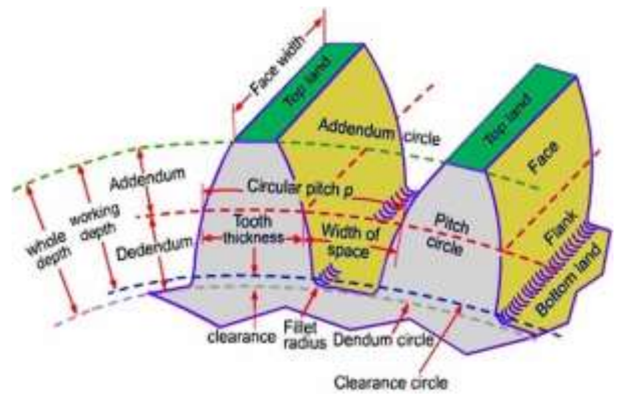

Figure 2: Gear geometry

The gear size is one of the most important variables in design optimization of gear set used in many applications [2].

Pitch circle diameter $d_P = m * z$

Addendum $h_a = m$

Dedendum $h_f = 1.157m$

Clearance $C = 0.157m$

Where m is the module

Planetary (Epicyclic) Gear train (which consists of sun, planet and ring gear)

z_s = No of teeth in sunwheel gear

z_p = No of teeth in planet gear

z_r = No of teeth in annulus ring gear

The three gear forms must share the same module *m to* mesh properly

$$d_{p\_r} = 2 * d_{p\_p} + d_{p\_s} \tag{1}$$

$$d_{r\_s} = d_{p\_s} - 2*h_f$$

$$d_{r\_p} = d_{p\_p} - 2*h_f$$

$$d_{r\_r} = d_{p\_r} + 2*h_f$$

$$d_{o\_s} = d_{p\_s} + 2*h_a$$

$$d_{o\_p} = d_{p\_p} + 2*h_a$$

$$d_{i\_r} = d_{p\_r} - 2*h_a$$

Where d_p pitch diameter, d_r root diameter, d_o outer diameter and d_i inner diameter

The subscripts s, p and r are used for sun, planet and ring gear respectively

Gears in gear boxes sometimes get short life due to wear and breakage by repetitive load during operation time [8], the two primary failure modes for gears [5] (as shown in Figure 3) are:

1) **Tooth Breakage** - from excessive bending stress, and

2) **Surface Pitting/Wear** - from excessive contact stress

Bending stress plays a significant role in gear design wherein its magnitude is controlled by the nominal bending stress and the stress concentration due to the geometrical shape. The bending stress is indirectly related to shape changes made to the cutting tool [13]

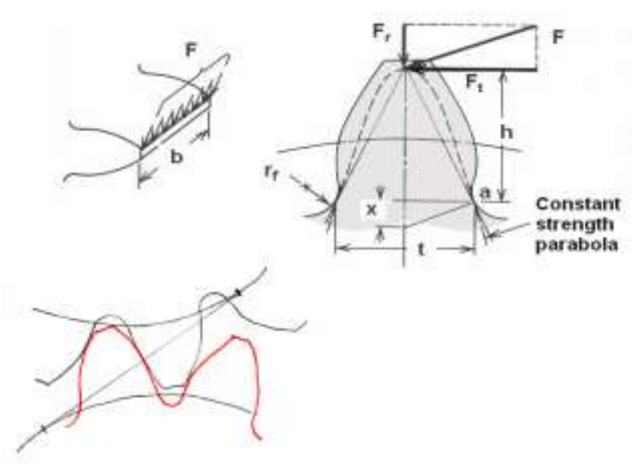

Figure 3: Bending and surface contact stresses on gear tooth

$$\sigma_b = \frac{F_t}{b*m*Y_j} * K_s * K_a * K_v * K_m * K_B * K_f \qquad (2)$$

Where F_t is the tangential force acting on the tip of the gear tooth, Y_j is approximately 0.34

$$K_v = \frac{50}{50+\sqrt{V}} \qquad \text{V in ft/min} \qquad (3)$$

Linear speed V is

$$V = \frac{\pi * d_{p\_s} * N_{in}}{60} \qquad (4)$$

$$\sigma_c = C_p \sqrt{\frac{F_t}{b*d_{p\_s}*I} * K_o * K_s * K_m * K_v} \qquad (5)$$

$$C_p = \sqrt{\frac{1}{\pi * \left(\frac{1-(\mu_s)^2}{E_s} + \frac{1-(\mu_p)^2}{E_p}\right)}} \qquad (6)$$

Where μ_s and μ_p are coefficient of friction for both sun and planet gears (the same material)

E_s and E_p are modulus of elasticity for both sun and planet gears (the same material)

$$T = \frac{60 * P_{in}}{2*\pi*N_{in}} \qquad (7)$$

$$F_t = \frac{T}{d_{o\_s}/2} \qquad (8)$$

$$\sigma_y = 0.75 * \sigma_u \qquad (9)$$

$$\sigma_{b\_all} = 0.6 * \sigma_y * \frac{Y_N}{K_R * SF} \qquad (10)$$

$$\sigma_{c\_all} = \sigma_y * \frac{Z_N * C_H}{K_R * SF} \qquad (11)$$

3. Design Aspects of the Planetary Gear Train

To calculate the total mass *M* of the planetary gear train, the masses of sun gear, planet gears and ring gear (M_s, M_p and M_r) are to be calculated as shown below:

Firstly the face area *A* and volume *V* are to be calculated

Sun Gear

$$A_s = \frac{\pi}{4}\left(d_{r\_s}{}^2 - d_{i\_s}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{o\_s}{}^2 - d_{r\_s}{}^2\right)\right) \quad (12)$$

$$M_S = V_S * \rho = A_S * b * \rho$$

Planet Gear

$$A_p = \frac{\pi}{4}\left(d_{r\_p}{}^2 - d_{i\_p}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{o\_p}{}^2 - d_{r\_p}{}^2\right)\right) \quad (13)$$

$$M_P = V_P * \rho = A_P * b * \rho$$

Ring Gear

$$A_r = \frac{\pi}{4}\left(d_{o\_r}{}^2 - d_{r\_r}{}^2\right) + \left(0.5 * \frac{\pi}{4}\left(d_{r\_r}{}^2 - d_{i\_r}{}^2\right)\right) \quad (14)$$

$$M_r = V_r * \rho = A_r * b * \rho$$

Total mass is:

$$M = M_s + M_p + M_r$$

Gear ratio of the planetary gearbox can be found from the following relation:

$$u = 1 + \frac{z_r}{z_s}$$

The mechanical properties of the different materials used in this study are presented in Table 1 and the values of data input for the optimization problem that can be assumed [3 and 12] as presented in Table2

Table 1: Mechanical Properties of different materials [14 and 15]

	Modulus of Elasticity (E)	**Ultimate Tensile Strength (σ_u)**	**Density ρ**	**Poisson's Ratio μ**
	GPa	Mpa	Kg/m^3	
Low Alloy Steel	209	600	7800	0.29
Stainless Steel	200	620	8000	0.27
Aluminum	72	257	2700	0.33
Plastic (Polyethylene PET)	2.5	50	1300	0.37

Table 2: Data Input

P_{in}	1.1, 2.2, 3, 4, 5.5	K_o	1.25
N_{in}	1500	Y_N	1
α	20	Z_N	1.1
Y_j	0.34	K_R	1
K_a	1	K_f	1
K_s	1	C_H	1
K_m	1.2	K_B	1
K_i	1	I	0.14
SF	1.5		

4. Genetic Algorithm in MATLAB Toolbox

Optimization is the process of finding the minimum or maximum value that a particular function attains and finding the value for the independent variables of objective function [16]. GA is a heuristic method, which is based on natural selection, the process that drives biological evolution. GA starts by generating a population of individuals in the space search. The choice of the size of population and the manner for representing the individual solutions are very important in order to promote the success of the method [11]. In fact, it is a series of random iterations and evolutionary computations which simulate the process of selection, crossover, and mutation occurred in natural selection and population genetic [12]

5. Optimization Model Formulation

The optimization model using MATLAB Toolbox routines consists of three files, objective function file, nonlinear constraints function file and genetic algorithm solver file.

The multi-objective optimization problem can be formulated as follows:

Find the design variables vector

$$x = \left(z_s, z_p, m, b, d_{i\_s}, d_{i\_p}, d_{o\_r}\right) \quad (15)$$

which minimizes the objective function

$$f = Min(M - u) \quad (16)$$

Subject to the behavior constraints:

$$\sigma_b \le \sigma_{b\_all} \quad (17)$$

$$\sigma_c \le \sigma_{c\_all}$$

$$6 \le \frac{b}{m} \le 12$$

$$z_p > z_s$$

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And the side constraints (limits in mm): $\vec{X}_L \leq \vec{X} \leq \vec{X}_U$

$$17 \leq z_s \leq 30 \qquad (18)$$

$$17 \leq z_p \leq 40$$

$$1 \leq m \leq 5$$

$$10 \leq b \leq 40$$

$$20 \leq d_{i\_s} \leq 40$$

$$20 \leq d_{i\_p} \leq 40$$

$$300 \leq d_{o\_r} \leq 700$$

z_s and z_p are integer

Generated heat, wear, creep, fatigue life and dynamic loadings are not considered in this research

6. Results and Discussions

A baseline design has been constructed to relate the optimum mass and gear ratio of the used materials. The baseline design is made of used material with its mechanical properties and the following variables assumptions

m_o = 3 mm, $Z_s)_o$ = 24, $Z_p)_o$= 30, $d_{is})_o$ = 30 mm, $d_{ip})_o$ = 40 mm and $d_{or})_o$= 400 mm

The gear tooth width has been calculated from the bending stress and then the mass is calculated for each input power value as shown in Table 3.

Gear ratio for all cases is u_o = 4.5.

Abbreviations for the materials used are:

Material Type	Abbreviation
Low Alloy Steel	I
Stainless Steel	II
Aluminum	III
Plastic Polyethylene PET	VI

Table 3: Baseline design output data

Material Type	Input Power KW	Tooth Width (b_o)	Gear Train Mass (M_o)
I	1.1	18	11.785
	2.2	18	11.785
	3	18	11.785
	4	18	11.785
	5.5	18	11.785
II	1.1	18	12.08765
	2.2	18	12.08765
	3	18	12.08765
	4	18	12.08765
	5.5	18	12.08765
III	1.1	18	4.07958
	2.2	18	4.07958
	3	18	4.07958
	4	18	4.07958
	5.5	18	4.07958
VI	1.1	18	1.9642
	2.2	18	1.9642
	3	24	2.597
	4	32	3.466

The results obtained from the optimization model have been collected in the next table, Table 4

Table 4: Optimization Model Results

Input Power (KW)	Material Type	Optimal Solutions			Gain %	
		$x_{opt} = (Z_S, Z_P, m, b, ds_S, ds$	$\hat{M}_{opt}$	$\hat{u}_{opt}$	$\hat{M}_{opt}$	$\hat{u}_{opt}$
1.1	I	(20, 36, 2, 19, 24, 40, 236)	0.330	1.244	67.0	24.4
	II	(18, 38, 2, 21, 21, 40, 241)	0.392	1.383	60.8	38.3
	III	(18, 27, 3, 19, 32, 40, 279)	0.453	1.111	54.7	11.1
	VI	(18, 32, 3, 18, 32, 40,317)	0.601	1.235	39.9	23.5
2.2	I	(19, 28, 3, 18, 35, 40, 290)	0.482	1.099	51.8	9.9
	II	(17, 26, 3, 20, 30, 40, 268)	0.455	1.124	54.5	12.4
	III	(26, 33, 3, 18, 40, 40, 354)	0.700	1.009	30.0	0.9
	VI	(23, 29, 3, 22, 40, 40, 313)	0.681	1.005	31.9	0.5
3	I	(22, 28, 3, 18, 40, 40, 302)	0.516	1.010	48.4	1.0
	II	(20, 28, 3, 19, 37, 40, 294)	0.520	1.067	48.0	6.7
	III	(26, 33, 3, 24, 40, 40 354)	0.934	1.009	6.6	0.9
	VI	(27, 34, 3, 21, 40, 40, 365)	0.684	1.004	31.6	0.4
4	I	(25, 32, 3, 18, 40, 40, 343)	0.681	1.013	31.9	1.3
	II	(23, 29, 3, 19, 40, 40, 313)	0.588	1.005	41.2	0.5
	III	x	x	x	x	x
	VI	(27, 34, 3, 28, 40, 40, 365)	0.684	1.004	31.6	0.4
5.5	I	(27, 34, 3, 21, 40, 40, 365)	0.905	1.004	9.5	0.4
	II	(26, 33, 3, 20, 40, 40,	0.8	1.0	19.	0.9

		354)	09	09	1	
	III	x	x	x	x	x
	VI	x	x	x	x	x

From the results obtained in the previous table, a bar graph has been drawn relating optimum mass gain with respect to input power for different gear materials as shown in Figure 4 below. Another figure (Figure 5) representing optimum gear ratio gain via input power for different materials has been drawn.

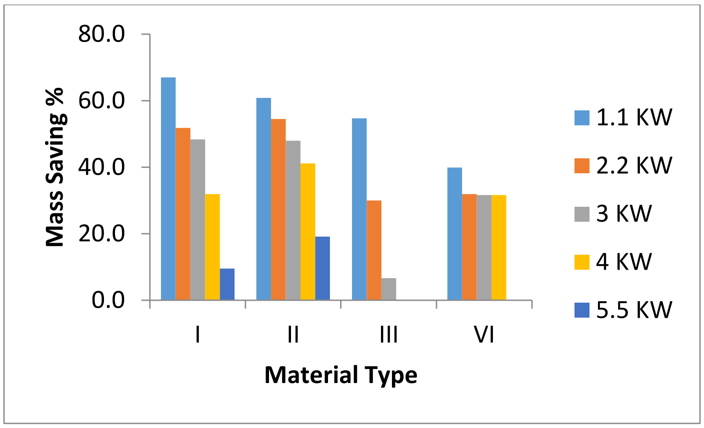

Figure 4: Optimum mass saving of planetary gear train related with input power for different materials

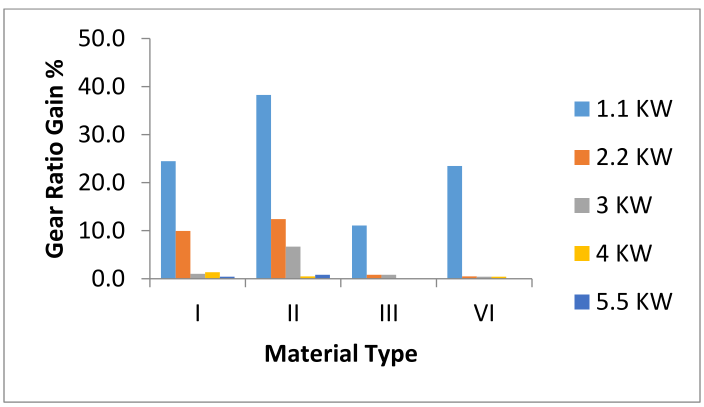

Figure 5: Optimum gear ratio gain relating with input power for different materials

Results obtained in Figure 4 have shown that for each material type studied in this research, the optimum mass saving is decreased with increasing the input power from 1.1 KW to 5.5 KW at the same input speed due to increasing the optimum mass.

Comparing the materials studied, it is found that stainless steel has relatively maximum output mass saving because it has maximum ultimate tensile strength and density while in case of plastic material it has relatively equal values because it has gradual increasing in the mass M_o. In case of alloy steel it has nearest values to stainless steel

From the gear ratio saving figure, it is observed the optimum gear ratio saving is decreased with increasing the input power from 1.1 KW to 5.5 KW at the same input speed due to decreasing the optimum gear ratio. In case of 1.1 KW input power it has maximum values of gear ratio saving compared to other power values.

7. Conclusions

A multi-objective optimization problem has been formulated using MATLAB optimization toolbox routings implementing genetic algorithm to obtain the minimal mass and maximum gear ratio of planetary gear train. Linear, nonlinear and side constraints have been formulated related to design variables.

Results obtained have shown that the optimization model succeeded in reaching the optimum values of design variables.

Results indicated that the optimum mass saving of planetary gear train has maximum values in case of stainless steel while has relatively equal values in case of plastic material. While optimum gear ratio has maximum values for 1.1 KW input power

This paper describes the multi-objective optimization technique which can be considered as a new research method in optimization; especially it is applied in planetary gear train.

Conflict of Interest

The authors declare no conflict of interest.

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